Bending strength of silica glass

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Tensile and bending tests are useful to characterize the mechanical behavior of ceramics. Theoretical comparisons between results of both tests are usually done based on Weibull statistics. In previous experiments on borosilicate glass, no agreement was found between experimental and theoretical values of the ratio of the maximum bending and tensile stresses at 50 percent probability of fracture. In this investigation, additional experiments in bending have been performed, to measure the distribution of fracture initiation points. Good agreement with theory is found. The previous disagreement could be attributed to fatigue effects.

1. Introduction

Strength of brittle materials is usually measured by means of either bending or tensile tests. This is a highly complex problem [1, 2]. The major difficulty with tensile testing is mounting the specimen in the machine. Fixtures used for tensile testing of ductile materials cannot be used, because they rely on plastic deformation of the specimen ends to assure gripping. If used on a brittle material, they cause fracture before any significant deformation occurs. On the other hand, dog-bone grips or other grips for use with button-ended specimens require careful adjustment to attain the near perfect alignment required to eliminate spurious bending stresses. Therefore, in brittle materials of relatively large size, it is easier, and more usual, to measure mechanical strength through bending tests.

On the other hand, specimens such as optical fibers, having small transverse dimensions (micrometers) are highly flexible. Thus, bending stresses are unimportant in them and tensile tests can be performed quite easily using, for example, a capstan-drive grip system [3]. By contrast, in these specimens, the conventional bending test is nearly impossible to perform, because the high flexibility demands such a small spacing between end supports.

Two assumptions commonly made in analyses of fracture in brittle materials are: that in the material some distribution of flaws exists initially; and that the fracture process consists of the propagation of a single crack from that surface flaw which, in relation to its size, is most highly stressed in tension. Compressive stresses are usually ignored. Based on these assumptions some sort of "weakest link" model is developed to describe the material. Although other models have been used, Weibull statistics [4, 5] are used most often because they fit a great many data sets.

In the present work, measurements of fracture initiation points and loads in bending are statistically analyzed to assess agreement with theoretical studies based in Weibull statistics.

2. Theoretical background

In testing cylindrical specimens of test length L the cumulative probability, P, of fracture is given by

$$P(\sigma^*) = 1 - \exp\left\{-L \int f(\sigma)\,\mathrm{d}s\right\} \tag{1}$$

Here σ^* is some stress value characterizing the load level in the test; f is a function of the local surface stress, σ; and the integration is performed over the specimen surface. The function $f(\sigma)$ for a two-parameter Weibull distribution [6] is given by

$$f(\sigma) = \frac{(\sigma/\sigma_0)^m}{(S_0 L_0)} \tag{2}$$

where, m is a material-dependent constant referred to as the Weibull exponent, and σ_0 (stress), S_0 (area), and L_0 (length) are normalization constants.

Consider a circularly cylindrical specimen of radius R and length $2a + L$ loaded in fourpoint bending by forces of magnitude F, as depicted in Fig. 1. Each end span has length a and the center span is L. The center span bending moment is Fa and the maximum stress is $\sigma_m = 4Fa/\pi R^3$. Within the center span, the stress anywhere on the specimen surface is independent of axial position and can be expressed as

$$\sigma = \frac{\sigma_m y}{R} = \sigma_m \sin\theta \tag{3}$$

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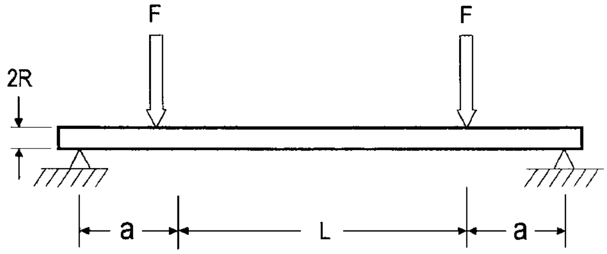

Figure 1 Specimen and test geometry for four point bending.

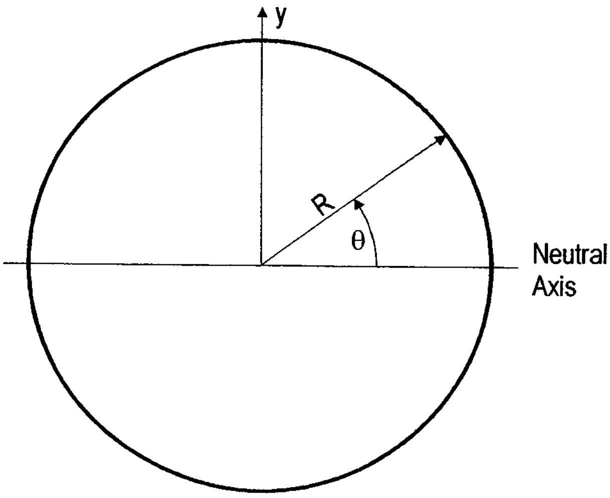

Figure 2 Cross section of circularly cylindrical specimen with dimensional notation.

Here y, the distance from the neutral axis, and θ, the angle from the neutral axis, are shown in Fig. 2. Because the stress is independent of axial position the integration in Equation 1 need only be performed over the circumference; and, because compressive stress has no effect, over only half of the circumference. Thus, using Equation 2 in (1) gives

$$P(\sigma_m) = 1 - \exp\left\{-\left(\frac{\sigma_m}{\sigma_0}\right)^m \left(\frac{RL^2}{S_0L_0}\right)\int_0^\pi \sin^m\theta\,\mathrm{d}\theta\right\} \tag{4}$$

Define the function $M(R, L, m)$ as $M = (RL^2/S_0L_0)\int_0^\pi \sin^m\theta\,\mathrm{d}\theta$ which is independent of stress. M is also independent of θ which is merely a dummy variable of integration over the prescribed range. Rearrange Equation 4 to the form $\exp\{-(\frac{\sigma_m}{\sigma_0})^m M\} = 1 - P(\sigma_m)$ and take the logarithm of each side. This gives

$$-\ln\{1 - P(\sigma_m)\} = \left(\frac{\sigma_m}{\sigma_0}\right)^m M \tag{5}$$

Take the logarithm of each side of Equation 5. The result can be arranged into an equation of the form

$$\ln Y = m \ln \sigma_m + C \tag{6}$$

Here, $Y = -\ln\{1 - P(\sigma_m)\}$ is one basic parameter for making a Weibull plot; the other is the logarithm of the measured load or stress σ_m. The slope of such a plot is the Weibull exponent, m. The final term C will depend upon the test geometry and upon m but will not depend upon the load (or stress).

The theoretical distribution of fracture angles, for a random distribution of surface flaws around the circumference, is called the fracture angle probability density, which is proportional to the derivative of $P(\sigma_m)$ with respect to θ. It is obtained first by differentiation of Equation 4

$$\begin{aligned}\frac{\mathrm{d}P(\sigma_m)}{\mathrm{d}\theta} &= \exp\left\{-\left(\frac{\sigma_m}{\sigma_0}\right)^m M\right\}\left(\frac{\sigma_m}{\sigma_0}\right)^m \frac{\mathrm{d}M}{\mathrm{d}\theta}\\ &= [1 - P(\sigma_m)]\left(\frac{\sigma_m}{\sigma_0}\right)^m \left(\frac{RL^2}{S_0L_0}\right)\sin^m\theta\\ &= B\sin^m\theta\end{aligned}$$

where B is a factor independent of angle. Therefore, the fracture angle probability density is given by

$$g(\theta) = A\sin^m\theta \tag{7}$$

Here A is another factor that is independent of angle, and it is obtained by requiring the integration of $g(\theta)$ over its range to be unity. Symmetry of the sine function allows the range to be compressed from 0 to π to 0 to $\pi/2$.

3. Experimental procedure

The material used in these experiments was borosilicate glass. The glass was obtained initially as rod stock of 6 mm diameter, approximately one meter in length. Bending test specimens were 50 mm in length. These were fabricated by scratching and breaking to length. The central span of the four-point loading jig was 20 mm. Loading and support "knife edges" were steel rods of 6.3 mm diameter.

After breaking to length, all fifty specimens were heat-treated to reduce residual stresses. Specimens were held at 500°C for 30 minutes. No attempt was made to assess possible devitrification during annealing. Tests were carried out in a universal testing machine, MTS model 810, using a ram speed of 4 μm/s.

The theoretical framework in which these results are being interpreted here assumes no variation in bending moment along the rod axis. This condition only prevails in the center span of the specimen, and so those few specimens that fractured in an end span were disregarded for the present purposes.

After completion of each bending test, the fracture surface of the sample was inspected using an optical microscope. This enabled us to determine the location of the initiation flaw and measure its angular position, θ, with respect to the neutral axis of bending. The experimental uncertainty of each angular measurement was plus or minus two degrees.

4. Results

Measurements of angle of fracture initiation are shown as a histogram in Fig. 3 (for larger than 90 degrees the supplement of the angle was used). The mean value of the angle obtained from experiments is 71 degrees (with a standard deviation of 14 degrees).

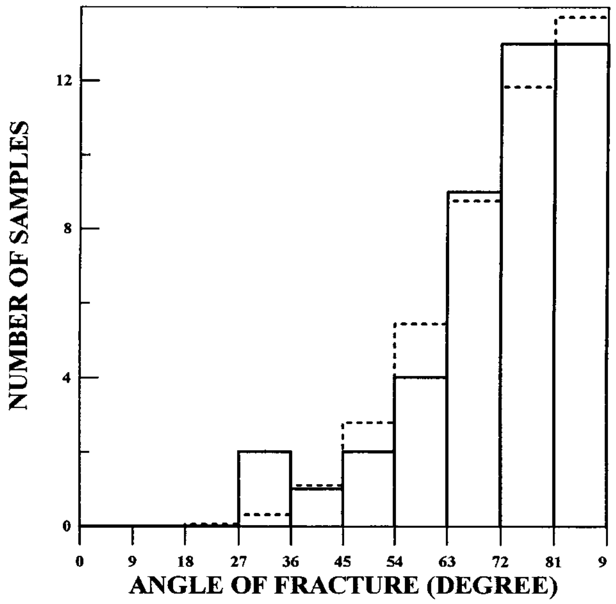

Figure 3 Histogram of the frequency versus fracture angle. Broken line represents the histogram calculated from the probability density function $g(\theta) = \underline{2.02}\ \sin^{5.9}\theta$.

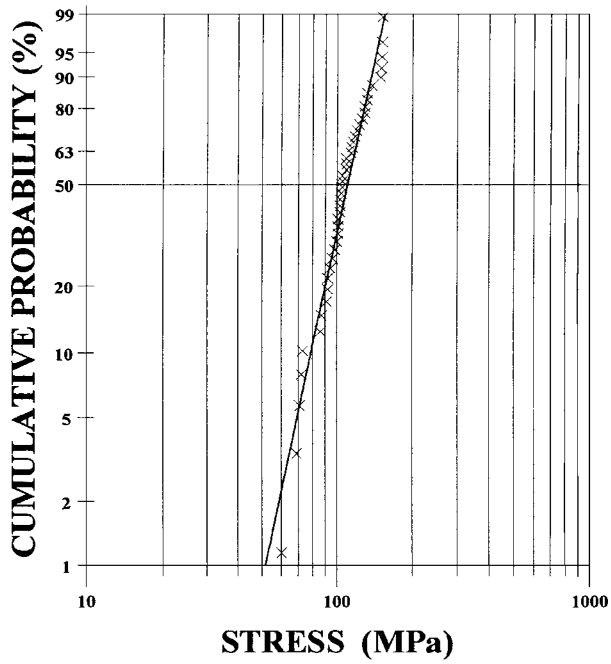

Figure 4 Weibull diagram for tensile and bending tests of 6-mm-diameter borosilicate glass.

Fig. 4 shows the Weibull diagram for the glass specimens. The number of broken specimens is forty-four. Cumulative probability was estimated by the rank function $P(i) = (i - 0.5)/n_t$, where i is the ordered observation and n_t the total number of data points (for each individual set $n_t = 22$). The same rank function was used to plot the fracture stress in the Weibull diagram and this function is a good estimator of the cumulative probability [7]. Least squares fitting of the data resulted in a value of the Weibull parameter $m = 5.9$ with a standard deviation of 0.2. At fifty percent probability of fracture the stress was 110 MPa with a standard deviation of 9 MPa.

5. Discussion

Comparison of experimental data with predictions of fracture angle probability density (Equation 7), which is based in Weibull statistics, can be done in two ways.

A theoretical histogram can be obtained from the density function. Each level is calculated by integration of this density function between lower and upper limits of the level. This histogram is represented in Fig. 3 as a broken line and shows good agreement with the experimental histogram already mentioned (full line).

Experimental mean fracture angle and standard deviation already indicated can be compared with the theoretical mean calculated from the assumed density distribution function. This mean is given by:

$$[\theta] = \int_0^{\pi/2} \theta g(\theta)\, d\theta \Big/ \int_0^{\pi/2} g(\theta)\, d\theta = 72 \text{ degrees}$$

with a standard deviation of

$$\text{SD} = \left\{ \int_0^{\pi/2} (\theta - [\theta])^2 g(\theta)\, d\theta \Big/ \int_0^{\pi/2} g(\theta)\, d\theta \right\}^{1/2}$$
$$= 13 \text{ degrees}$$

These values compare very well with the experimental values (71 and 14 degrees).

On the other hand, comparison of theoretical calculations with other experimental results on tensile and bending strengths shows some discrepancies. Difficulties may arise when comparing results from the different tests. For instance, proof testing experiments generally are used as a guarantee of mechanical strength for optical fibers. Proof testing may be conducted either by tensile [8] or by bending [9] tests, and there are definite problems in trying to establish an appropriate correlation between the two. The main difference between the two is the stress distributions on their respective cross sections. In the tensile test, the stress is constant over the specimen cross section, whereas in the bending test, it varies from the maximum tensile stress at the lowermost fibers, decreasing along the surface (and through the bulk) to zero at the neutral axis and to compression above the neutral axis.

Based upon circularly cylindrical geometry and Weibull statistics, Medrano and Gillis [10] and Kittl and Diaz [11] have calculated the theoretical ratio of tensile stress σ_t to maximum bending stress σ_m that results in equal fracture probabilities in both tests. Call this geometry dependent ratio T. Previous experimental measurements on glass were made [12] to assess these theoretical calculations. Quite poor agreement was found between theory and experiments, and it was concluded that fracture in the materials tested was not satisfactorily described by Weibull statistics.

Experimental Weibull parameters and stress values for 50 percent fracture probability are given in Table I for the three sets of tests: prior tensile and bending tests and the current bending tests. For the previous results the experimental ratio T was 0.46 while the theoretical value was significantly higher: 0.64. If the maximum bending stress for 50 percent probability of fracture

TABLE I Characteristic material parameters obtained from tensile and bending tests of rods of borosilicate glass

Weibull Parameter, m	Stress at 50 Percent Probability of Fracture (MPa)
6.2 (0.2)	38 (6) tensile (previous)
6.5 (0.3)	82 (12) bending (previous)
5.9 (0.2)	110 (9) bending (present)

Numbers in Parentheses denote standard deviations obtained by least-squares fitting of data sets.

of the present experiments is taken, together with the tensile stress of previous experiments, the T value is 0.38, even lower than previous value.

One possible explanation for the increasing discrepancy is that although the materials used in the previous and present experiments have the same heat treatment, the samples were taken from two different material batches. Therefore, they could have different intrinsic strengths.

Another possible explanation is based upon the well known but not satisfactorily quantified phenomenon that for ceramic materials an increase of the rate of stressing produces an increase in fracture stress. This stress rate dependence is called fatigue. Unfortunately, in our previous experiments, ram speeds were not recorded, because the influence of stress rate was not considered. An explanation of the disagreement of experimental and theoretical values of the parameter T could be attributed to fatigue, but to reach any conclusion, additional experiments should be done.

Any stress rate effect is additionally complicated by a fundamental difference between tensile and bending tests. In tensile tests the stress is uniform and, therefore, the stress rate is also uniform throughout the specimen. However, in the bending test the stress is variable, as indicated in Equation 3, and then the stress rate is also variable. Thus, equal ram speeds in bending tests do not assure equal stress rates at the fracture initiation location for all samples.

If differences in stress-rate are not large, corresponding differences in fracture stress are practically negligible. In bending the ratio between minimum and maximum stress-rates is approximately two (based on a minimum observed fracture angle of approximately 30 degrees). The ratio between two fracture stresses is related to the stress-rate ratio $(\mathrm{d}\sigma/\mathrm{d}t)$ by [13] $\sigma_1/\sigma_2 = \{\mathrm{d}\sigma_1/\mathrm{d}t)/(\mathrm{d}\sigma_2/\mathrm{d}t)\}^{\{1/(n+1)\}}$, where n is the fatigue parameter. For the borosilicate glass tested at 50% RH (relative humidity) n is about 31 [14] and, thus, the maximum fracture stress is only about 2% larger than the minimum fracture stress when the rate ratio is two.

A way to avoid the influence of fatigue is to use approximately the same stress-rate for tensile and bending tests. A rough approach would be to take equal tensile and maximum bending stress-rates. A better approach is to use instead of the maximum stress-rate, the mean stress-rate in bending $[\mathrm{d}\sigma/\mathrm{d}t]$ given by

$$\left[\frac{\mathrm{d}\sigma}{\mathrm{d}t}\right] = \left(\frac{\mathrm{d}\sigma_m}{\mathrm{d}t}\right)[\sin\theta] \qquad (8)$$

where $\mathrm{d}\sigma_m/\mathrm{d}t$ is the maximum bending stress-rate and $[\sin\theta]$ is the mean of $\sin\theta$ is calculated by using the density of probability $g(\theta)$ as

$$[\sin\theta] = \int_0^\pi \sin\theta g(\theta)\,\mathrm{d}\theta \bigg/ \int_0^\pi g(\theta)\,\mathrm{d}\theta$$

$$= \left\{\Gamma\left(\frac{m}{2}+1\right)\right\}^2 \bigg/ \left\{\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)\cdot\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)\right\} \qquad (9)$$

where m is the Weibull exponent and Γ is the gamma function. In our experiment ($m = 5.9$), the mean stress-rate is 0.93 of the maximum stress-rate.

Of course the tensile tests must be performed first and analyzed so that the value of m is available to determine the corresponding bending test stress rate.

6. Conclusions

1. Experimental measurements of angular positions of fracture initiation show excellent agreement with calculated distributions based upon Weibull statistics.

2. Disagreement of experimental and theoretical values for the ratio of tensile stress to maximum bending stress that results in equal fracture probabilities in both tests could be related to fatigue; but additional experiments must be done to confirm this assumption.

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